Решение на задачите за домашна работа

1. а) $(x^2-3)^3-(x-2)(x^2+4)(x+2)-x^4.(x^2-10)=$
$=\cancel{x^6}-9\cancel{x^4}+27x^2-27-\cancel{x^4}+16-\cancel{x^6}+10\cancel{x^4}=27x^2-11;$
б) $(2x-3)^3-(2-3x)^3-35x^3+90x^2=$
$=8\cancel{x^3}-36\cancel{x^2}+54x-27-8+36x-54\cancel{x^2}+27\cancel{x^3}-35\cancel{x^3}+90\cancel{x^2}=$
$=90x-35.$

2. а) $(2a-b)(2a+b)(4a^2+b^2)=16a^4-b^4$
$(4a^2-b^2)(4a^2+b^2)=16a^4-b^4$
$16a^4-b^4=16a^4-b^4;$
б) $8(a-1)^3=a^3+3a^2(a-2)+3a(a-2)^2+(a-2)^3$
$8(a^3-3a^2+3a-1)=$
$=a^3+3a^3-6a^2+3a(a^2-4a+4)+a^3-6a^2+12a-8$
$8a^3-24a^2+24a-8=5a^3-12a^2+12a-8+3a^3-12a^2+12a$
$8a^3-24a^2+24a-8=8a^3-24a^2+24a-8.$

3. а) $4(x-6)-x^2(2+3x)+x(5x-4)+3x^2(x-1)=$
$=4x-24-2x^2-3x^3+5x^2-4x+3x^3-3x^2=-24;$
б) $2y(y^2-1)-(3-y)^3+(y^2+2)(y+1)-(4y^3+27y-8y^2)=$
$=2y^3-2y-27+27y-9y^2+y^3+y^3+y^2+2y+2-4y^3-27y+8y^2=-25.$

4. а) $(2x-0,5).(0,5+2x)-(-3x-1)^2+(2x-0,5)^2+x^2=$
$=4x^2-0,25-9x^2-6x-1+4x^2-2x+0,25+x^2=-8x-1;$
б) $(1+2x)^3-(2x-1)^3+(x-1)(x^2+x+1)+(-x-1)(x^2-x+1)=$
$=1+6x+12x^2+8x^3-8x^3+12x^2-6x+1+x^3-1-x^3-1=24x^2.$

5. $A=-4P-(-2Q-2P-P+Q)=-4P+3P+Q=Q-P$
$A=a^2-2ab+b^2-a^2-2ab-b^2=-4ab$
$A=-4ab.$

6. а) $(-2y+4z).(-5x)-((3x-2y).2z-5y(4x-2z))=$
$=10xy-20xz-(6xz-4yz-20xy+10yz)=$
$=10xy-20xz-6xz-6yz+20xy=$
$=30xy-26xz-6yz;$

б) $by-(y-b)(y+b)+b(y-b)=by-y^2+b^2+by-b^2=2by-y^2$;
в) $(y-3).y-(6y^2-12y):6y=y^2-3y-y+2=y^2-4y+2$.

7. $V=\dfrac{32y^4x^6-8y^2x^5}{-2x+8x^2y^2}=\dfrac{8y^2x^5(4y^2x-1)}{2x(4xy^2-1)}=4y^2x^4,\ -2x+8x^2y^2\neq 0.$

8. $B=\left(\dfrac{1}{2}-\dfrac{1}{4}x\right).4x-(-6x^2+3x):(-3x)=2x-x^2-2x+1=1-x^2.$

$x=\dfrac{(-2)^2.(-3)^4}{(-3)^2.2^3}.\left(-\dfrac{2}{3}.1\dfrac{1}{2}\right)^{10}=\dfrac{2^2.3^4}{2^3.3^2}.\left(-\dfrac{2}{3}.\dfrac{3}{2}\right)^{10}=\dfrac{9}{2}.$

$B=1-\left(\dfrac{9}{2}\right)^2=1-\dfrac{81}{4}=\dfrac{4-81}{4}=-\dfrac{77}{4}=-19\dfrac{1}{4}.$

9. а) $43^2=(40+3)^2=1600+240+9=1849$;
б) $51^2=(50+1)^2=2500+100+1=2601$;
в) $78^2=(80-2)^2=6400-320+4=6084$;
г) $45^2=(40+5)^2=1600+400+25=2025$.

10. $\left(2x+\dfrac{1}{2}\right)^2-\left(3x+\dfrac{1}{3}\right)^2-\left(\dfrac{2}{3}-2x\right).\left(\dfrac{2}{3}+2x\right)+x^2+3.\left(-\dfrac{1}{6}\right)^2=$

$=4x^2+2x+\dfrac{1}{4}-9x^2-2x-\dfrac{1}{9}-\dfrac{4}{9}+4x^2+x^2+\dfrac{1}{12}=$

$=9x^2-9x^2+\dfrac{1}{4}-\dfrac{5}{9}+\dfrac{1}{12}=\underbrace{\dfrac{1}{4}-\dfrac{5}{9}+\dfrac{1}{12}}_{36}=\dfrac{9-20+3}{36}=-\dfrac{8}{36}=-\dfrac{2}{9}.$

11. а) $2x^2+(x+5)^2-2(x+7)^2=2(3x-72,5)+(x-6)^2$
$2x^2+x^2+10x+25-2(x^2+14x+49)=6x-145+x^2-12x+36$
$3x^2+10x+25-2x^2-28x-98=x^2-6x-109$
$x^2-x^2-12x=73-109,\ -12x=-36,\ x=3$;
б) $2x+(x-1)^3-x^3=-3x^2$
$2x+x^3-3x^2+3x-1-x^3=-3x^2$
$5x=1,\ x=\dfrac{1}{5}$.

12. $(2x)^2-4=4x^2-4=4(x^2-1)=4(x+1)(x-1)$ се дели на 4.

13. а) $\left(-\frac{4}{5}x^7-\frac{2}{3}y^m z^2\right).\left(-\frac{4}{5}x^7+\frac{2}{3}y^m z^2\right)=$

$=-\left(\frac{4}{9}y^{2m}z^4-\frac{16}{25}x^{14}\right)=\frac{16}{25}x^{14}-\frac{4}{9}y^{2m}z^4;$

б) $296.304=(300-4)(300+4)=90\,000-16=89\,984.$

14. $A=a(x^2+3x+4)-ax(x+3)-2(2x-15)=$

$=a\cancel{x^2}+3a\cancel{x}+4a-a\cancel{x^2}-3a\cancel{x}-4x+30=4(a-x)+30.$

При $a=x-3$ $A=4(x-3-x)+30=-12+30=18.$

15. а) $\left(x-\frac{1}{2}\right)^3-\left(x-\frac{1}{2}\right).\left(x^2+\frac{1}{2}x+\frac{1}{4}\right)+\frac{3}{2}x^2=$

$=\cancel{x^3}-\frac{3}{2}\cancel{x^2}+\frac{3}{4}x-\cancel{\frac{1}{8}}-\cancel{x^3}+\cancel{\frac{1}{8}}+\frac{3}{2}\cancel{x^2}=\frac{3}{4}x;$

б) $\left(x+\frac{1}{4}\right).\left(x^2-\frac{1}{4}x+\frac{1}{16}\right)-x\left(x-\frac{1}{8}\right)^2-\left(\frac{1}{2}x-1\right).\left(\frac{1}{2}x+1\right)=$

$=x^3+\frac{1}{64}-x\left(x^2-\frac{1}{4}x+\frac{1}{64}\right)-\frac{1}{4}x^2+1=$

$=\cancel{x^3}+\frac{1}{64}-\cancel{x^3}+\frac{1}{4}\cancel{x^2}-\frac{1}{64}x-\frac{1}{4}\cancel{x^2}+1=$

$=\frac{1}{64}-\frac{1}{64}x+1=1\frac{1}{64}-\frac{1}{64}x.$

16. а) $M=(ax-1)^2-x(x+2a)+x^2=$

$=a^2x^2-2ax+1-\cancel{x^2}-2ax+\cancel{x^2}=a^2x^2-4ax+1;$

б) При $a=\frac{1}{2}$ и $x=4$ за M получаваме

$M=\left(\frac{1}{2}\right)^2.16-4.\frac{1}{2}.4+1=\frac{1}{4}.16-8+1=-3.$

$M=-3.$

17. а) $M.N=(x^2+2)(x^2-2)=x^4-4;$

б) $M^2-N^2=(x^2+2)^2-(x^2-2)^2=$

$=(x^2+\cancel{2}+x^2-\cancel{2})(\cancel{x^2}+2-\cancel{x^2}+2)=8x^2;$

в) $M^2+N^2=(x^2+2)^2+(x^2-2)^2=x^4+\cancel{4x}+4+x^4-\cancel{4x}+4=$

$=2x^4+8;$

г) $M^3 + N^3 = (x^2+2)^3 + (x^2-2)^3 =$
$= x^6 + 6x^4 + 12x^2 + 8 + x^6 - 6x^4 + 12x^2 - 8 = 2x^6 + 24x^2;$
д) $M^3 - N^3 = (x^2+2)^3 - (x^2-2)^3 = x^6 + 6x^4 + 12x^2 + 8 - x^6 + 6x^4 - 12x^2 + 8 = 12x^4 + 16.$

18. а) $(4x+3y).(16x^2 - 12xy + 9y^2) = 64x^3 + 27y^3;$
б) $(4x-5y)(16x^2 + 20xy + 25y^2) = 64x^3 - 125y^3.$

19. а) $A = \dfrac{x}{0,3} - \dfrac{1}{2}(2x+6) + \dfrac{x}{-3} = \dfrac{10x}{3} - x - 3 - \dfrac{x}{3} = 2x - 3;$

б) $B = \left(\dfrac{3}{2}x - 1\right)^2 - (1,5x-2)\left(\dfrac{3}{2}x + 2\right) =$
$= \dfrac{9}{4}x^2 - 3x + 1 - \dfrac{9}{4}x^2 + 4 = 5 - 3x;$
в) $C = (2+x)^2 - 2(1-x)^2 + (-x+2)^2 =$
$= 4 + 4x + x^2 - 2(1 - 2x + x^2) + (2-x)^2 =$
$= 4 + 4x + x^2 - 2 + 4x - 2x^2 + 4 - 4x + x^2 = 6 + 4x;$
г) $D = \dfrac{5}{4}\left(x^2 - \dfrac{4}{5}\right) - \left(\dfrac{1}{2}x - 3\right)\left(3 + \dfrac{1}{2}x\right) - 11 =$
$= \dfrac{5}{4}x^2 - 1 - \dfrac{1}{4}x^2 + 9 - 11 = x^2 - 3;$
е) $E = x(x-5)^2 + 4x(2x-3) - (x-2)^3 - (-2x)^2 =$
$= x(x^2 - 10x + 25) + 8x^2 - 12x - (x^3 - 6x^2 + 12x - 8) - 4x^2 =$
$= x^3 - 10x^2 + 25x + 8x^2 - 12x - x^3 + 6x^2 - 12x + 8 - 4x^2 = x + 8.$

20. $B = 3y^2\left(\dfrac{1}{8}y + x\right) - \dfrac{1}{2}y^3(2y-1) + xy(4-3y) + (y^2-2)(y^2+2) =$
$= \dfrac{3}{8}y^3 + 3xy^2 - y^4 + \dfrac{1}{2}y^3 + 4xy - 3xy^2 + y^4 - 4 = \dfrac{7}{8}y^3 + 4xy - 4$
$x = 3;$
$y^2 - 2y + 1 - 2 = y^2 + 3 \Rightarrow -2y = 4 \Rightarrow y = -2$
$B = \dfrac{7}{8}(-2)^3 + 4.3.(-2) - 4 = -7 - 24 - 4 = -35.$

21. $2(a^4 - 2a^2 + 1) - a^4 + 9 - \dfrac{1}{2}(2a^4 + 3a^2 + 2a^3 + 3a - 8a^2 - 12) =$

$$= 2a^4 - 4a^2 + 2 - a^4 + 9 - a^4 - \frac{3}{2}a^2 - a^3 - \frac{3}{2}a + 4a^2 + 6 =$$
$$= -a^3 - \frac{3}{2}a^2 - \frac{3}{2}a + 17 = -\left(-\frac{1}{2}\right)^3 - \frac{3}{2}\left(-\frac{1}{2}\right)^2 - \frac{3}{2}\left(-\frac{1}{2}\right) + 17 = 17\frac{1}{2}.$$

22. $\left\{\frac{\text{о}}{4}[(\text{б}+\text{и})^2 - (\text{б}-\text{и})^2].\frac{\text{ч}}{4}[(\text{а}+\text{м})^2 - (\text{а}-\text{м})^2]\right\}$ + те = обичам + те.

23. Умножаваме двата многочлена и получаваме

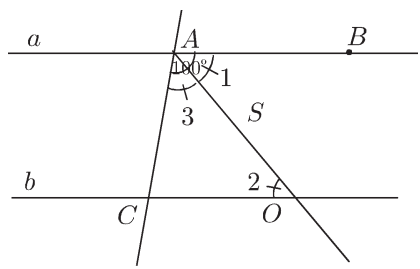
$(x^2 + px - 3)(x^2 - 5x + 13) =$
$= x^4 + px^3 - 3x^2 - 5x^3 - 5px^2 + 15x + 13x^2 + 13px - 39 =$
$= x^4 + (p-5)x^3 + (10 - 5p)x^2 + (15 + 13p)x - 39.$
За да не съдържа x^2 полученият многочлен, трябва коефициентът пред него да е нула, т.е. $10 - 5p = 0 \Rightarrow p = 2$.

24. Извършваме умножението и привеждаме многочлена в нормален вид. Получаваме

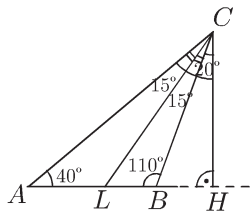
$$x^5 + (a-1)x^4 + (b - a + 3)x^3 + (3a - b - 1)x^2 + (1 + 3b)x - 3.$$

За да не съдържа полученият многочлен x^4 и x^3 трябва едновременно да са изпълнени следните две условия $a - 1 = 0$ и $b - a + 3 = 0$. От първото равенство получаваме $a = 1$ и заместваме с $a = 1$ във второто равенство за да намерим b: $b - 1 + 3 = 0 \Rightarrow b = -2$.

25. $\sphericalangle CAB = \frac{5}{9}$ от $180^\circ = 100^\circ$.
$\sphericalangle 1 = \sphericalangle 3 = \frac{1}{2} \sphericalangle CAB = 50^\circ$.
$(a \| b) \cap AO$
$\sphericalangle 1 = \sphericalangle 2 = 50^\circ$ (кръстни).



26. Нека $\sphericalangle ACB = x \Rightarrow \sphericalangle CAB = x + 10°$.
$\sphericalangle ABC = \sphericalangle CAB + 70° = x + 10° + 70° = x + 80°$.
За $\triangle ABC$: $\sphericalangle ABC + \sphericalangle ACB + \sphericalangle CAB = 180°$ (Т за сбор от ъглите в триъгълник)
$x + 80° + x + x + 10° = 180°$,
$3x = 90° \Rightarrow x = 30°$



$\Rightarrow \sphericalangle ACB = 30°; \sphericalangle CAB = 40°; \sphericalangle ABC = 110° \Rightarrow \triangle ABC$ е тъпоъгълен. Построяваме CL – ъглополовяща и CH – височина.

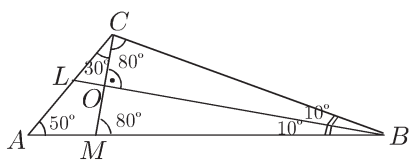
$\sphericalangle ABC$ е външен за $\triangle BHC \Rightarrow \sphericalangle ABC = \sphericalangle BHC + \sphericalangle BCH = 110° = 90° + \sphericalangle BCH \Rightarrow \sphericalangle BCH = 20° \Rightarrow \sphericalangle LCH = \sphericalangle LCB + \sphericalangle BCH = 15° + 20° = 35°$.

27. $\sphericalangle ACM = 60\%$ от $\sphericalangle CAB = \dfrac{60}{100}.50° = 30°$.

$\sphericalangle CMB = \sphericalangle CAM + \sphericalangle ACM$ (Т за външен ъгъл за $\triangle AMC$)

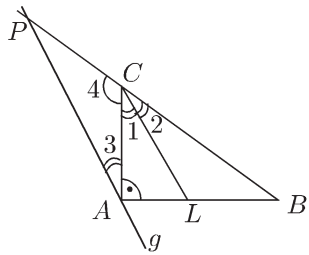
$\Rightarrow \sphericalangle CMB = 80°$.

Нека $CM \cap BL = 0$.
$\triangle OMB$ – правоъгълен;
$\sphericalangle MBO = 90° - \sphericalangle OMB = 10°$ (Т за сбор от острите ъгли в правоъгълен триъгълник)
$\sphericalangle ABL = \sphericalangle LBC = 10°$ (Свойство на ъглополовящата) $\Rightarrow \sphericalangle ABC = 20°$.



От $\triangle OBC$: $\sphericalangle OCB = 80°$ (Т за сбор на острите ъгли в правоъгълен триъгълник) $\Rightarrow \sphericalangle ACB = 30° + 80° = 110°$.

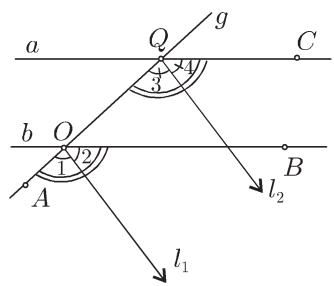
28. Нека $\sphericalangle 1 = \sphericalangle 2 = \alpha$.



$\sphericalangle 4 = \sphericalangle ACB + 80^\circ = 2\alpha + 80^\circ$;
$\sphericalangle PCA + \sphericalangle ACB = 180^\circ$ (Т за съседни ъгли), $2\alpha + 80^\circ + 2\alpha = 180^\circ$, $4\alpha = 100^\circ \Rightarrow \alpha = 25^\circ$;
$\sphericalangle 1 = \sphericalangle 2 = 25^\circ; \sphericalangle 4 = 2\alpha + 80^\circ = 130^\circ$; $(g \| CL) \cap AC$ $\sphericalangle 3 = \sphericalangle 1 = 25^\circ$ (кръстни).

От $\triangle PAC$: $\sphericalangle CPA = 180^\circ - (\sphericalangle PAC + \sphericalangle PCA) = 180^\circ - (25^\circ + 130^\circ) = 25^\circ$.

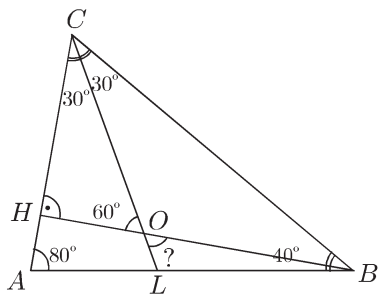
29. $(l_1 \| l_2) \cap g$



$\Rightarrow \sphericalangle 1 = \sphericalangle 3$ като съответни ъгли, но
$\sphericalangle 1 = \sphericalangle 2$ (свойство на ъглополовящата l_1)
$\sphericalangle 3 = \sphericalangle 4$ (свойство на ъглополовящата l_2)
$\Rightarrow \sphericalangle 1 = \sphericalangle 2 = \sphericalangle 3 = \sphericalangle 4$
$\Rightarrow \sphericalangle 1 + \sphericalangle 2 = \sphericalangle 3 + \sphericalangle 4$

$\Rightarrow \sphericalangle AOB = \sphericalangle OQC$
$\sphericalangle AOB$ и $\sphericalangle OQC$ са съответни ъгли при $(a; b) \cap g \Rightarrow a \| b$.

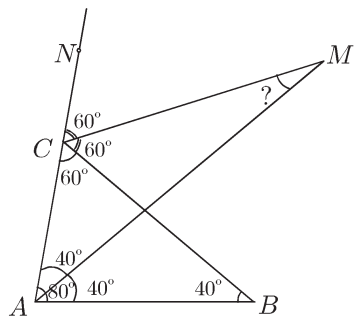
30. а) Търсим ъгъла между BH и CL.



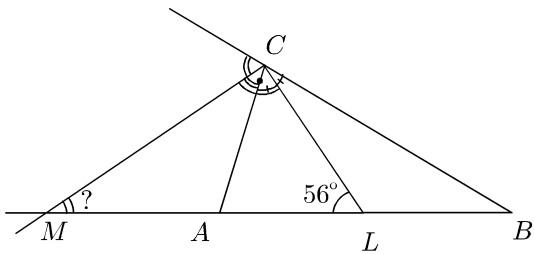
Под ъгъл между две прави се разбира острият ъгъл между тях.
$\sphericalangle A : \sphericalangle B : \sphericalangle C = 4 : 2 : 3$. Нека 1 равна част от отношението е $x \Rightarrow \sphericalangle A = 4x$; $\sphericalangle B = 2x$; $\sphericalangle C = 3x \Rightarrow \sphericalangle A + \sphericalangle B + \sphericalangle C = 180^\circ$ (Теорема за сбор от ъглите в $\triangle ABC$),
$4x + 2x + 3x = 180^\circ$; $x = 20^\circ \Rightarrow \sphericalangle A = 4x = 80^\circ$; $\sphericalangle B = 2x = 40^\circ$; $\sphericalangle C = 3x = 60^\circ$.

CL е ъглополовяща на $\sphericalangle ACB \Rightarrow \sphericalangle ACL = \sphericalangle LCB = 30^\circ$.

От $\triangle HOC : \sphericalangle CHO = 90°; \sphericalangle HCO = 30° \Rightarrow \sphericalangle HOC = 60°$
$\Rightarrow$ ъгълът между BH и CL е $60°$.
б) От $\sphericalangle A : \sphericalangle B : \sphericalangle C = 4 : 2 : 3 \Rightarrow$
$\Rightarrow \sphericalangle A = 80°; \sphericalangle B = 40°; \sphericalangle C = 60°$.
CM е ъглополовяща на външния ъгъл при върха C
$\Rightarrow \sphericalangle NCM = \sphericalangle MCB = 60°$
$\sphericalangle CAM = \sphericalangle MAB = 40°$ (свойство на ъглополовящата AM). За $\triangle CAM : \sphericalangle MCA + \sphericalangle CAM + \sphericalangle AMC = 180°$; $120° + 40° + \sphericalangle AMC = 180° \Rightarrow \sphericalangle AMC = 20°$.

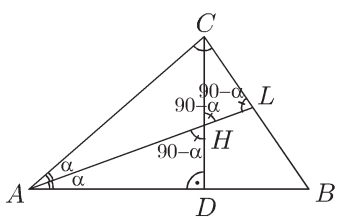


31. Използваме задача 26: CM и CL са ъглополовящи на вътрешен и външен ъгъл при върха $C \Rightarrow CM \perp CL$, т.е. $\sphericalangle MCL = 90°$.



От $\triangle MLC : \sphericalangle CML = 90° - \sphericalangle MLC = 90° - 56° = 34°$ (Теоремата за сбор от острите ъгли на правоъгълен триъгълник).

32. Нека $\sphericalangle CAL = \alpha \Rightarrow \sphericalangle LAB = \alpha$
(свойство на ъглополовящата).
От $\triangle ADH$ – правоъгълен
$\Rightarrow \sphericalangle AHD = 90° - \alpha$.
$\sphericalangle CHL = \sphericalangle AHD = 90° - \alpha$ (върхни),
но $\sphericalangle CHL = \sphericalangle HLC$ (по условие)
$\Rightarrow \sphericalangle HLC = 90° - \alpha$.



Разглеждаме $\triangle ALC$: $\sphericalangle CAL + \sphericalangle ALC + \sphericalangle ACL = 180°$,
$\alpha + 90 - \alpha + \sphericalangle ACL = 180° \Rightarrow \sphericalangle ACL = 90°$.

33. а) $(5c^2+cd+d^2)(3cd-3d^2-c^2)-(3cd-c^2-d^2)(5c^2+cd-3d^2)=$
$15c^3\not{d}+3c^2\not{d^2}+3c\not{d^3}-15c^2d^2-3c\not{d^3}-3d^4-5\not{c^4}-c^3\not{d}-c^2d^2-15c^3\not{d}+5\not{c^4}+$
$5c^2d^2-3c^2d^2+c^3\not{d}+cd^3+9cd^3-3c^2\not{d^2}-3d^4=-14c^2d^2-6d^4+10cd^3$;
б) $(-2+x)(x^2+4)-(3-x^2)^3-(x^2+1)x+9x^4-x^6=-2x^2-8+x^3+$
$4x-(27-27x^2+9x^4-x^6)-\not{x^3}-x+9x^4-x^6=-2x^2-8+3x-27+$
$27x^2-9\not{x^4}+\not{x^6}+9\not{x^4}-\not{x^6}=25x^2+3x-35$;
в) $\left(-\frac{1}{2}y-1\right)\left(\frac{1}{2}y-1\right)-y\left(y-\frac{1}{2^3}\right)^2+\left(y+\frac{1}{4}\right)\left(y^2+\frac{1}{16}-\frac{1}{4}y\right)=$
$-\left(\frac{1}{2}y+1\right)\left(\frac{1}{2}y-1\right)-y\left(y^2-\frac{1}{4}y+\frac{1}{64}\right)+y^3+\left(\frac{1}{4}\right)^3=-\left(\frac{1}{4}y^2-1\right)-$
$\not{y^3}+\frac{1}{4}y^2-\frac{1}{64}y+\not{y^3}+\frac{1}{64}=-\frac{1}{4}\not{y^2}+1+\frac{1}{4}\not{y^2}-\frac{1}{64}y+\frac{1}{64}=\frac{65}{64}-\frac{1}{64}y=$
$-\frac{1}{64}y+\frac{65}{64}$.

34. $x=1, y=\frac{|-2|.(-1)^8}{|-3|:|3|}=2$. $A=\frac{1}{8}\not{x^3}+\frac{3}{2}x^2y+6xy^2+8y^3-8x^3+$
$6x^2y-\frac{3}{2}xy^2+\frac{1}{8}y^3+7\frac{7}{8}x^3-7\frac{1}{2}x^2y=4,5xy^2+8\frac{1}{8}y^3=4,5.1.2^2+\frac{65}{8}.2^3=$
$18+65=83$.

35. а) $x(2-y)+y(y-2)=-x(y-2)+y(y-2)=(y-2)(y-x)$;
б) $(y-2z)(12y^2-7yz)-yz(2z-y)=(y-2z)(12y^2-7yz)+yz(y-2z)=$
$(y-2z)(12y^2-7yz+yz)=(y-2z)(12y^2-6yz)=6y(y-2z)(2y-z)$;
в) $a^2b^2-ab^2-ab-a^2=a^2(b^2-1)-ab(b+1)=a^2(b+1)(b-1)-ab(b+1)=$
$a(b+1)[a(b-1)-b]=a(b+1)(ab-a-b)$;
г) $m^2n^2-4n^2+m^2-4-2n(m^2-4)=n^2(m^2-4)+(m^2-4)-2n(m^2-4)=$
$(m+2)(m-2)(n^2+1-2n)=(m+2)(m-2)(n-1)^2$;
д) $9(2a-x)^2-4(3a-x)^2=[3(2a-x)]^2-[2(3a-x)]^2=(6a-3x-$
$6a+2x)(6a-3x+6a-2x)=-x(12a-5x)=x(5x-12a)$;
е) $\underbrace{b^2+6b+9}-25c^2=(b+3)^2-(5c)^2=(b+3+5c)(b+3-5c)$;
ж) $x^2-6\frac{1}{4}=x^2-\frac{25}{4}=x^2-\left(\frac{5}{2}\right)^2=\left(x+\frac{5}{2}\right)\left(x-\frac{5}{2}\right)$;

з) $x^2+2x-24 = \underbrace{x^2+2x+1}-25 = (x+1)^2-(5)^2 = (x+1+5)(x+1-5) = (x+6)(x-4)$;

и) $x^2-20x+91 = \underbrace{x^2-20x+100}-9 = (x-10)^2-3^2 = (x-10+3)(x-10-3) = (x-7)(x-13)$;

к) $6x^2+5x-6 = 6\left(x^2+\frac{5}{6}x-1\right) = 6\left(x^2+2.\frac{5}{12}x+\frac{25}{144}-\frac{169}{144}\right) = 6\left[\left(x+\frac{5}{12}\right)^2-\left(\frac{13}{12}\right)^2\right] = 6\left(x+\frac{5}{12}+\frac{13}{12}\right)\left(x+\frac{5}{12}-\frac{13}{12}\right) = 6\left(x+\frac{18}{12}\right)\left(x-\frac{18}{12}\right) = 6\left(x+\frac{3}{2}\right)\left(x-\frac{2}{3}\right)$.

36. а) Опростяваме лявата страна на тъждеството.

$90b^2-35b^3-(2-3b)^3+(2b-3)^3 = 90b^2-35b^3-(8-36b+54b^2-27b^3)+8b^3-36b^2+54b-27 = 90\cancel{b^2}-35\cancel{b^3}-8+36b-54\cancel{b^2}+27\cancel{b^3}+8\cancel{b^3}-36\cancel{b^2}+54b-27 = 90b-35 = 5(18b-7)$;

б) $1+2(-a-2b)+(-a-2b)^2 = (1-2b-a)^2$.

Лявата страна на тъждеството е формула за съкратено умножение: $[1+(-a-2b)]^2 = (1-a-2b)^2$.

37. За целта трябва да докажем, че стойността на израза B не зависи от стойността на променливата y. Привеждаме израза в нормален вид:

$B = 2(-2y-1)^2-(4y+3)^2+8y(y+2) = 2(4y^2+4y+1)-(16y^2+24y+9)+8y^2+16y = 8\cancel{y^2}+8\cancel{y}+2-16\cancel{y^2}-24\cancel{y}-9+8\cancel{y^2}+16\cancel{y} = -7$.

38. а) $(12x+5)^2-(8x-1)^2-(10x+7)(8x+3) = 78$

$144x^2+120x+25-(64x^2-16x+1)-(80x^2+30x+56x+21) = 78$

$144\cancel{x^2}+120x+25-64\cancel{x^2}+16x-1-80\cancel{x^2}-86x-21 = 78$

$50x+3 = 78$

$50x = 75$

$x = \frac{75}{50} = \frac{3}{2} = 1,5$;

б) $(9x-4)^2+(40x+1)^2-(41x-2)(41x+2) = 25$

$81x^2-72x+16+1600x^2+80x+1-[(41x)^2-4] = 25$

$81\cancel{x^2} + 8x + 17 + 1600\cancel{x^2} - 1681\cancel{x^2} + 4 = 25$
$8x + 21 = 25, 8x = 4, x = \frac{1}{2}$;
в) $9x(3-x) + (x-3)(x^2+9+3x) = 1 + (x-3)^3$
$27x - 9x^2 + x^3 - 27 = 1 + x^3 - 9x^2 + 27x - 27$
$0x = 1$, няма решение.

39. а) За да се дели изразът B на 22, то той трябва да се дели на 2 и на 11. Разлагаме B на множители: $B = 11x^2 - 99x + 220 = 11(x^2 - 9x + 20) = 11(x^2 - 4x - 5x + 20) = 11[x(x-4) - 5(x-4)] = 11(x-5)(x-4)$. По условие x е цяло число, следователно $x-5$ и $x-4$ са цели последователни числа, т.е. едно от тях е четно и се дели на 2.
б) $11x^2 - 99x + 220 = 0$. Използвахме разлагането от подточка а), т.е. $11(x-5)(x-4) = 0 \Rightarrow x = 5, x = 4$.

40. а) $\underbrace{\frac{6x+7}{7} + x = \frac{80+4x}{5} + \frac{30-2x}{-2}}_{70}$
$10(6x+7) + 70x = 14(80+4x) - 35(30-2x)$
$60x + 70 + 70x = 1120 + 56x - 1050 + 70x$
$130x + 70 = 70 + 126x$
$4x = 0, x = 0$;
б) $\underbrace{\frac{1+\frac{x}{4}}{2} - \frac{\frac{7x}{2}+1}{-6} - \frac{1+5x}{24} + \frac{\frac{7}{2}+6x}{-12} = \frac{1}{3}}_{24}$
$12\left(1 + \frac{x}{4}\right) + 4\left(\frac{7}{2}x + 1\right) - 1 - 5x - 2\left(\frac{7}{2} + 6x\right) = 8$
$12 + 3x + 14x + 4 - 1 - 5x - 7 - 12x = 8$
$17x - 17x = 0, 0x = 0$, всяко x е решение;
в) $2y - \frac{7y+3}{0,8} = \frac{3y+2}{-0,5} + \frac{3y+1}{-2} \Big| \cdot \frac{1}{10}$
$\underbrace{\frac{2y}{10} - \frac{7y+3}{8} = -\frac{3y+2}{5} - \frac{3y+1}{20}}_{40}$
$8y - 5(7y+3) = -8(3y+2) - 2(3y+1)$
$8y - 35y - 15 = -24y - 16 - 6y - 2$

$-27y - 15 = -30y - 18$
$3y = -3$
$y = -1.$